A Novel Application of Multi-objective Evolutionary Algorithm for Practical Optimizations through Simple Formulation and Post-optimal Evolution

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Abstract. To cope with many difficult problems that must be solved certainly for sustainable development goals, practical approaches available for rational decision making is highly demanded in modern technologies. In such situation, various optimization methods have been successfully applied so far. Particularly, under multiple goals someone of which conflict with each other, a field known as multi-objective optimization problem (MOP) has been studied from various aspects. Among them, multi-objective evolutionary algorithms (MOEAs) are especially interested in these decades. They are viewed as a useful technique for revealing a wide relation of objective function values among the conflicting objectives and supporting multi-objective optimization. To enhance its ability, in this paper, we have proposed a simple procedure for solving single-objective optimization problems using MOEA and the idea will be applied to some multi-objective optimization based on weighing and ε-constrained (lexicographic) approaches. Being classic, they are often used in various situations even presently due to the effectiveness regardless of their simplicity. Moreover, a post-optimal evolution is proposed for repairing some shortcomings inherent to those approaches and makes them more practical and adaptive. Actually, it is deployed in co-operation with our elite-induced multi-objective evolutionary algorithms. In the numerical experiment, a set of benchmark problems and the classical MOPs have been solved to examine the performance as global and practical optimization technique, respectively. Eventually, the proposed idea makes MOEA more useful in various decision-making environments encountered these days.

Keywords: Optimization, Multi-objective evolutionary algorithm, Problem formulation, Post-optimal evolution

1. INTRODUCTION

To cope with many difficult problems that must be solved certainly for sustainable development goals, practical optimization methods are highly demanded for supporting rational decision-making in modern technologies. In this sense, meta-heuristic optimization methods opened a new horizon since they can work with various situations flexibly and effectively. They never need differential information of functions at all and go well with meta-model or model of model. Noticing the amazing progress of simulation technique as in software and computer as in hardware, such feature is quite suitable for practical optimization.

Moreover, the idea has successfully extended to the area associated with multi-objective optimization problem (MOP). Actually, multi-objective evolutionary algorithms (MOEA) are especially interested in these decades (Coello, 2012). They are viewed as a multi-objective analysis that aims at revealing a wide relation on objective function values among the conflicting objectives and supporting multi-objective optimization. To expand availability of such MOEA, we first proposed (Yoo and Shimizu, 2018) a simple procedure for solving single-objective optimization problem (SOP) using MOEA and applied it to the real world optimization problem incorporated with a multi-objective optimization method known as $MOON^2$ (Shimizu and Kawada, 2002).

After confirming its solution ability more in detail, in this paper, we provide a portable technique that makes everyone easily engage in multi-objective optimization. For this purpose, we concern with certain scalarlized MOPs given as weighed and ε-constraint approaches. Being classic, they are often used even presently due to the effectiveness regardless of their simplicity. Then, a post-optimal evolution is proposed to repair some shortcomings inherent to those classic approaches. Actually, it is deployed in co-operation with our elite-induced multi-objective evolutionary algorithm (EI-MOEA). Finally, we discuss on the effectiveness of the proposed idea through solving a set of benchmark problems and then move on the classic MOP and its post-optimal evolution.

The rest of this section is organized as follows. Section 2 describes the proposed idea and its cool application. In Section 3, the effectiveness is verified through a few numerical experiments. Some conclusions are given in Section 4.

2. PROPOSED IDEA WITH A COOL APPLICATION

2.1 Multi-objective Optimization in Terms of Scalarlization

In general, MOP is described as follows.

$$\text{(p.1)}\quad \text{Min}\quad \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), \ldots, f_N(\boldsymbol{x})\}$$

$$\textit{subject to}\ \ \boldsymbol{x} \in X = \left\{ \boldsymbol{x} \,\middle|\, \begin{array}{ll} g_i(\boldsymbol{x}) \le 0, & (i = 1, \ldots, m1) \\ h_i(\boldsymbol{x}) = 0, & (i = 1, \ldots, m2) \end{array} \right\}$$

where $\boldsymbol{x}$ denotes a decision variable vector, X a feasible region and $\boldsymbol{f}$ an objective function vector, some elements of which conflict with one another.

The aim of this problem is to obtain a unique preferentially optimal solution through subjective judgments of decision maker (DM) on his/her preference. On the other hand, to reveal a certain tradeoff relation among the conflicting objectives and to provide useful information about the DM's preference is called as multi-objective analysis (MOA; Psarras, et al., 1990, Bennett, 1989, Sohpos, et al., 1980). Regardless of such fundamental definition, multi-objective evolutionary algorithm (MOEA) as a method for MOA has been developed under the name of "optimization". However, every MOEA is still useful for MOP since it can derive Pareto front as the essence of tradeoff readily and effectively.

As a popular approach for MOP, some scalarlized methods have been applied traditionally due to their simplicity in application. They try to transform the original MOP into SOP by a certain procedure. In the case of weighting and ε -constraint methods, this formulation is given as (p.2) and (p.3), respectively.

$$\text{(p.2)}\quad \underset{\boldsymbol{x}}{\text{Min}}\ \ V(f(\boldsymbol{x})) = \sum_{i=1}^{N} w_i f_i(\boldsymbol{x})\ \ \textit{subject to}\ \ \boldsymbol{x} \in \mathrm{X}$$

$$\text{(p.3)}\quad \underset{\boldsymbol{x}}{\text{Min}}\quad V(f(\boldsymbol{x})) = f_i(\boldsymbol{x})\ \ \textit{subject to}$$

$$\boldsymbol{x} \in \mathrm{X}\ \ \&\ \ f_j \le \varepsilon_j, \forall j \ne i$$

where w_i denotes the weighting coefficient representing the relative importance of the i-th objective and ε_j *the* upper bound compromise for j-th objective function. Generally speaking, however, there exist no ways to appropriately decide these preference parameters beforehand. That is an inherent weakness that should be overcome by these approaches.

2.2 Simple Formulation to Solve Scalarlized MOP by MOEA

Here, we propose an idea to solve SOP by using MOEA. That is very simple and deployed in terms of the following proposition.

Proposition: Objectives Min $V(\boldsymbol{f}(\boldsymbol{x}))$ and Max $V(\boldsymbol{f}(\boldsymbol{x}))$ always conflict with each other.

This means problem "(p.4) Min $\{V(\boldsymbol{f}(\boldsymbol{x})), -V(\boldsymbol{f}(\boldsymbol{x}))\}$ s.t. $\boldsymbol{x}\in$X" or "(p.4') Min $\{V(\boldsymbol{f}(\boldsymbol{x})), 1/V(\boldsymbol{f}(\boldsymbol{x}))\}$ s.t. $\boldsymbol{x}\in$X" is viewed as a bi-objective problem. Accordingly, we can solve any SOP by MOEA as follows.

(1) Apply a certain MOEA for the above (p.4) or (p.4').
(2) Select the solution with the minimum value of $V(\boldsymbol{f}(\boldsymbol{x}))$ as the preferentially optimal solution of (p.1).

Thus, we can obtain the preferentially optimal solution of the original problem by MOEA. In terms of such idea, we can solve MOP after transforming it into the scalar one as

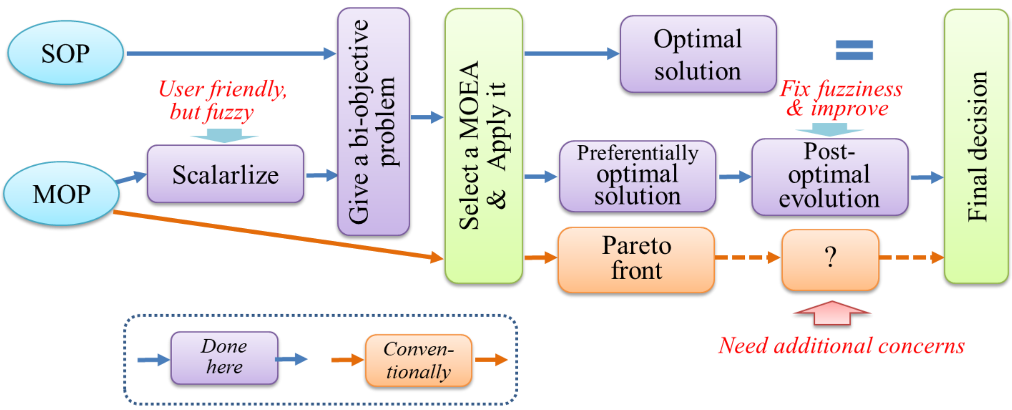

Fig.1 Framework of the proposed idea compared with conventional stance of MOEA: Though the conventional application is limited to just derive the Pareto front, the proposed procedure has practical and wide possibilities. What's more, all we need is just an appropriate MOEA.

mentioned in the previous subsection. This seems to be a cool application that can extend the availability of MOEA.

2.3 Post-optimal Evolution Using Elite-induced MOEA

It is possible to cope with the inherent weakness of pre-determined value function or scalarlized approach through a post-optimal evolution. For this purpose, our elite-induced MOEA (EI-MOEA) (Shimizu, Takayama and Ohishi, 2012) is just amenable. The principle behind this idea is just simple and straightforward from the original MOEA. Instead of using all randomly generated initial solutions, it introduces some elite solutions that are obtained from the prior solution and just apply an appropriate MOEA. We can expect the existence of the elite solutions will induce the Pareto front at the direction toward their pre-existing locations. By adjusting the number and location of such elites, it is possible to manipulate a distribution of final solutions so that the result would lie on a specific region on Pareto front. Moreover, due to the existence of the elites, selection pressure that might contribute to the accuracy and convergence speed is always kept at high level. This makes the algorithm powerful and computation load smaller.

Since the present aim is to obtain the preferentially optimal solution, the distribution should be limited around it. Hence, this attempt is realized by the following formulation.

$$\text{(p.5)}\quad Min\,\{f_1(\boldsymbol{x}),..,f_N(\boldsymbol{x})\}\ \text{subject to}$$
$$\boldsymbol{x}\in X\ \&\ \sum_{i=1}^{N}(1-f_i(\boldsymbol{x})/f_i^*)^2\le\delta$$

where f_i^* denotes the *i*-th objective value of the optimal solution of (p.2) or (p.3) and δ an upper bound extent of post-optimal evolution (Shimizu et al., 2016).

As a summary of this subsection, in Fig.1, we show a framework concerned by the proposed idea to make its significance clear. Just by an appropriate MOEA, we can cope with a variety of interests in engineering optimization regardless whether it is given as SOP or MOP. Eventually, the proposed idea is promising to expand and enhance the availability of MOEA greatly.

3. NUMERICAL EXPERIMENTS

3.1 Evaluation with Various Benchmark Problems

Now, to evaluate the global solution ability through comparison with other methods, we solved ten popular benchmark problems some of which have multiple peaks of objective functions (Refer to Appendix). We deployed our approach taking NSGA-II (Deb et al., 2000) and compared its performance to one conventional direct search N-M (Nelder & Mead) and four popular evolutionary methods such as DE (Differentially Evolution), PSO (Particle Swarm Optimization), GA (Genetic Algorithm) and SA (Simulated Annealing). Each problem was solved using the algorithms in R with the respective default tuning parameters (Library or code names are for NSGA-II: nsga2 in "mco"; DE: "DEoptim"; PSO: "pso"; GA: "rbga in genalg"; SA and N-M: in "optim"). On the other hand, we set the population size (*popsz*) and generation time (*gener*) depending on the dimension of decision variables *D* as Eqs.(1) and (2), respectively. In terms of the known optimal value *f*opt, we evaluated the success numbers by Eq, (3).

$$popsz = \min(10D, 60) \tag{1}$$

$$gener = \min(100 * popsz^{0.7}, 2000) \tag{2}$$

$$Success\#:\ \text{if}\ (|f\text{opt} - f(\boldsymbol{x})| < eps*(1.0 + |f\text{opt}_{|}|)\ ),$$
$$\text{then}\ Success\# = +1,\ \text{where}\ eps=1.0\text{E-}2 \tag{3}$$

Every problem was solved 31 times and we show only a part of these results in Tables 1 and 2 due to the space limitation. Under the present conditions, only DE could get full mark all over the problems, and only two problems (De Jong & Martin/ Gaddy) are solved correctly by all methods. To evaluate the performance among the methods quickly, we ranked the method according to the Success# as shown in Table 3.

Table 1 Comparison among the methods for 4-D Rosenbrock (*D*=4, popsz=40, gener=1322, *f*opt=0)

Item	Min	Median	Mean	Max	Suc#
DE	0.0	0.0	0.0	0.0	31
PSO	1.08E-4	7.81E-4	9.02E-4	2.24E-3	31
GA	9.90E-3	6.02E-1	5.78E-1	1.2438	1
N-M	3.04E-7	1.48E-5	7.17E-1	3.7074	25
SA	6.86E-3	2.95E-2	2.99E-2	8.07E-2	2
NSGA2	3.70E-3	6.59E-1	1.00236	3.8230	2

Table 2 Comparison among the methods for Griewangk (*D*=10, popsz=60, gener=1756, *f*opt=0)

Item	Min	Median	Mean	Max	Suc#
DE	0.0	0.0	0.0	0.0	31
PSO	0.0	0.0	9.58E-4	7.40E-3	31
GA	1.02E-6	7.40E-3	4.54E-3	7.40E-3	31
N-M	2.93E-3	3.37E-2	7.44E-2	3.67E-1	3
SA	6.59E-1	8.88E-1	8.74E-2	9.90E-1	0
NSGA2	1.00E-4	7.54E-3	1.08E-2	4.37E-2	26

Despite the poor performance of SA as of meta-heuristic algorithm, conventional N-M has a favorable feature if we notice its simple algorithm. Though the proposed NSGA-II is inferior to DE and PSO, it outperforms the rests including the relative method like GA.

Moreover, four poor results regarding NSGA-II are analyzed more in detail. We showed their histogram based on the objective value in Fig. 2. From all of those results, the solution ability of the proposed approach is known to be satisfactory except for 4d-Rosenbrock and Griewangk problems.

Finally, we can conclude the total performance ranks at the third place following DE and PSO a bit behind. From these, we can claim the proposed NSGA-II is comparable to the conventional evolutionary methods and sufficient even as a global optimization method for SOP.

3.2 Post-optimal Evolution to Enhance Prior Solution of Scalarlized MOP

Though the weighting and ε-constraint methods belong to classical method, they are often used due to the effectiveness regardless of their simplicity as mentioned already. Here, through the proposed NSGA-II, we solved the bi-objective FES1 benchmark (Huband et al., 2006, Appendix) formulated as SOP, i.e., (p.2) or (p.3). We showed those results in the "*Before*" column in Table 4.

As popularly known, shortcomings of those classical methods refers to the stiff setting of preference parameters like weighting and ε constraint values. Generally speaking, it is almost impossible to pre-determine those values appropriately. Hence, it makes sense to re-evaluate the result after such plain optimization. For example, taking three cases shown "Method" and "Preference parameter" in Table 4, i.e., case ε_2 =0.1 weighs f_2 more than f_1 and case ε_2 =0.7 oppositely more f_1 while case "Weighting" balances both, we carried out this post-optimal evolution following the procedure mentioned in Sec.2.3.

Using the elite-induced NSGA-II under the conditions *popsz*=6, *gener*=200 and single elite (original optimal solution) shown in "*Before*" column, we obtain the result for each case as shown in "*After*" column. Among them, we used bold face to show the best solution while underline infeasible ones. By virtue of the post-optimal evolution, let us note some solutions shown by red letters outperform the original solution.

Table 3 Summary of ranking of each method (Number in brackets denotes the *Success#* defined by Eq.(3))

Problem	1st	2nd	3rd	4th	5th	last
Shekel's fox hole	DE, NSGA2, GA [31]			PSO [29]	N-M [2]	SA [1]
Schwefel	DE [31]	PSO[27]	NSGA2[22]	GA [9]	SA [3]	N-M [2]
De Jong	All solved successfully					
Goldstein/Price	DE, PSO, GA [31]			NSGA2[30]	SA [28]	N-M [16]
Branin	DE,PSO,GA,N-M,SA [31]					NSGA2[30]
Martin/ Gaddy	All solved successfully					
Rosenbrock	DE, PSO, SA, N-M [31]				NSGA2[23]	GA [16]
4-D Rosenbrock	DE, PSO [31]		N-M [25]	NSGA2 [2]	SA [2]	GA [1]
Hyper sphere	DE,PSO,NSGA2,GA,N-M[31]					SA [1]
Griewangk	DE, PSO, GA [31]			NSGA2[26]	N-M [3]	SA [0]

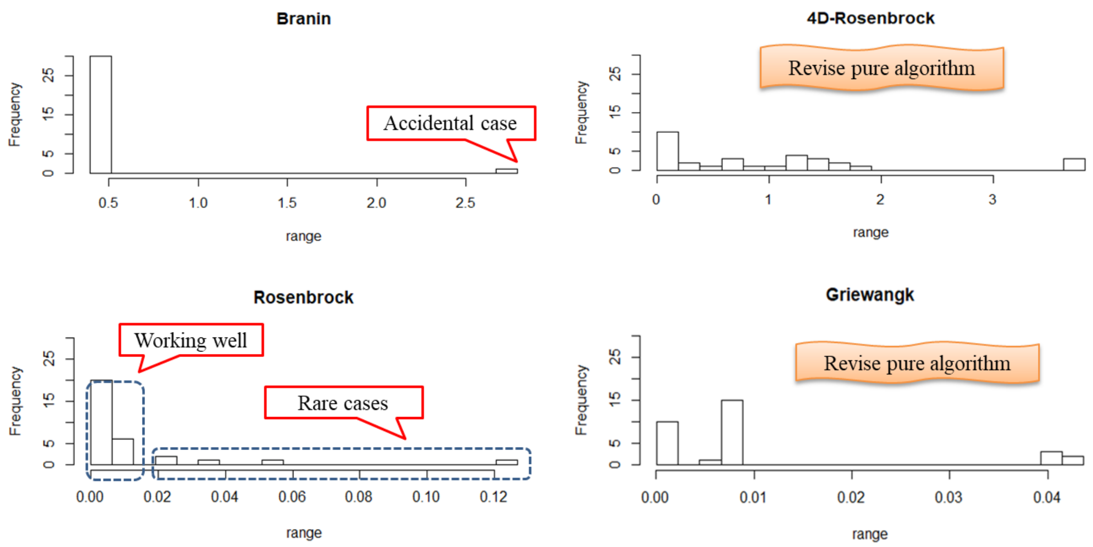

Fig.2 Histogram of success number for lower rank results regarding NSGA-II (Range means objective value).

Among those, we might find out more favorable solution by elaborately inspecting those solutions again. In fact, since the pre-determined ε constraint values are not strictly definite, it is meaningful to work on this re-evaluation over the underlined infeasible solutions. For example, as seen in the case of ε_2 =0.1, an infeasible (2.5548, 0.1086) might be more preferable to the best (2.7178, 0.0875) or the elite (2.6366, 0.1) since allowing a slight violation of ε constraint on f_2 (<0.1) can give a big return on f_1.

In a summary, in Fig.3, we describe the results of the original optimizations (w12, e<0.1 & e<0.7) and the post-optimal evolutions (post-w12, post<0.1 & post<0.7). (In reference, we add the Pareto front derived by ordinal NSGA-II under the conditions *popsz*=20, *gener*=100). We

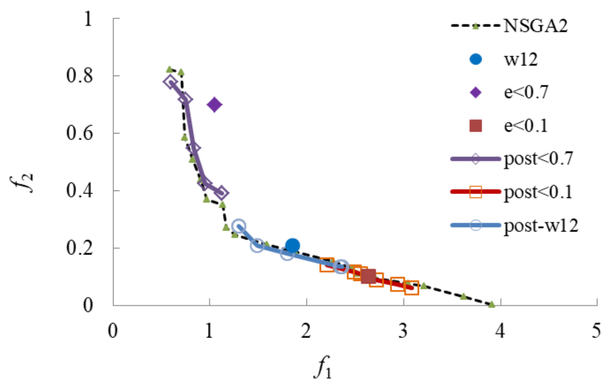

Fig.3 Post-optimal evolution for three optimal solutions obtained by classical methods.

know every post-optimal evolution can build correctly its corresponding segment of the Pareto front. Through inspecting the front around there, we might improve the quality of multi-objective optimization and have a chance to update the prior decision rationally. Eventually, we can relax the shortcomings of the classical methods and make them more flexible and adaptive approach.

4. CONCLUSION

To make a rational decision in various fields, we have successfully resolved some difficult problems not only through SOP but also MOP. Especially, MOEAs are being interested in these decades. Though they are useful techniques for multi-objective analysis, in this paper, we have proposed a simple procedure for solving SOP by MOEA to enhance its usefulness. Moreover, the idea has been applied to the classic multi-objective optimization under the weighing and ε-constraint approaches and deployed as a post-optimal evolution that aims at repairing shortcomings inherent to those methods. Actually, it is developed in a co-operation with our elite-induced multi-objective evolutionary algorithms. After all, the proposed idea can make an analysis method like MOEA be available for an effective and practical solution method of SOP and certain MOPs as well.

To examine the effectiveness of the proposed idea, a set of benchmark problems have been solved by the proposed NSGA-II and compared with other methods. Then, through the post-optimal evolution for the classical MOPs, we have shown its significance in practical decision-making. In future studies, it is interesting to compare the performance among the other MOEAs like MODE, MOPSO, etc.

Table 4 Result of FES1 benchmark by the proposed procedure mentioned in Sec. 2.1

Method	Preference parameter	Before (original)		After (post-optimal)	
		(f_1, f_2) : elite	V^*	(f_1, f_2)	V^*
Weighting	$\boldsymbol{w}$ = $(0.2, 0.8)^T$	(1.8510, 0.2077)	0.5363	**(1.4875, 0.2089)**	**0.4646**
				(1.2923, 0.2749)	0.4784
				(1.7935, 0.1795)	0.5023
				(2.3434, 0.1357)	0.5772
				(2.3603, 0.1347)	0.5798
ε-constraint	ε_2 =0.1 (weigh on f_2)	(2.6366, 0.1)	2.6366	(2.2045, 0.1406)	2.2045
				(2.4921, 0.1168)	2.4921
				(2.5548, 0.1086)	2.5548
				(2.7178, 0.0875)	**2.7178**
				(2.9363, 0.0738)	2.9363
				(3.0837, 0.0588)	3.0837
	ε_2 =0.7 (weigh on f_1)	(1.0531, 0.7)	1.0531	(0.5971, 0.7773)	0.5971
				(0.7454, 0.7163)	0.7454
				(0.8292, 0.5473)	**0.8292**
				(0.9483, 0.4234)	0.9483
				(1.1198, 0.3913)	1.1198

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Appendix. Benchmark Problems

1. Shekel's fox hole: *f*opt =0.998004 at (-32.0, -32.0)

$$\frac{1}{f(\boldsymbol{x})}=0.002+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6};\ -50\le x_j\le 50,\ \forall j$$

$$a_{ij}=\{-32,-16,0,16,32,-32,-16,0,16,32,-32,\ -16,0,16,32,-32,-16,0,16,32,-32,-16,0,16,32;\ -32,-32,-32,-32,-32,-16,-16,-16,-16,-16,\ 0,0,0,0,0,16,16,16,16,16,32,32,32,32,32\}$$

2. Schwefel: *f*opt =0.0 at (418.9829, …, 418.9829)

$$f(\boldsymbol{x})=-\sum_{j=1}^{6}\sin(\sqrt{|x_j|});\ -500\le x_j\le 500,\ \forall j$$

3. De Jong: *f*opt =3905.93 at (1, 1)

$$f(\boldsymbol{x})=3905.93+100(x_1^2-x_2)^2+(1-x_1)^2;\ -2\le x_j\le 2,\ \forall j$$

4. Goldstein & Price: *f*opt =3 at (0, -1)

$$f(\boldsymbol{x})=\{1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\}\ *\{30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\};\ -2\le x_j\le 2,\ ,\forall j$$

5. Branin: *f*opt =0.3977272 at (-22/7, 12.275), (22/7, 2.275) or (66/7, 2.475)

$$f(\boldsymbol{x})=a(x_2-bx_1^2+cx_1-d)^2+e(1-f)\cos(x_1)+e;\ -5\le x_j\le 10,\ \forall j$$

$$a=1, b=\frac{5.1}{4}(\frac{7}{22})^2,\ c=\frac{35}{22},\ d=6,\ e=10,\ f=\frac{7}{176}$$

6. Martin & Gaddy: *f*opt =0 at (5, 5)

$$f(\boldsymbol{x})=(x_1-x_2)^2+\{(x_1+x_2-10)/3\}^2;\ 0\le x_j\le 10,\forall j$$

7. Rosenbrock : *f*opt =0 at (1, 1)

$$f(\boldsymbol{x})=100(x_1^2-x_2)^2+(1-x_1)^2;\ -2\le x_j\le 2,\ \forall j$$

8. 4-D Rosenbrock: *f*opt =0 at (1, 1, 1, 1)

$$f(\boldsymbol{x})=\sum_{i=1}^{3}\{100(x_i^2-x_{i+1})^2+(1-x_i)^2\};\ -2\le x_j\le 2,\forall j$$

9. Hyper sphere: *f*opt =0 at (0, 0, 0, 0, 0, 0)

$$f(\boldsymbol{x})=\sum_{j=1}^{6}x_j^2;-6\le x_j\le 6,\forall j$$

10. Griewangk: *f*opt =0 at (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

$$f(\boldsymbol{x})=1+\sum_{i=1}^{10}\frac{x_i^2}{4000}-\prod_{i=1}^{10}\cos(\frac{x_i}{\sqrt{i}});\ -5\le x_j\le 5,\forall j$$

FES1: $Min\{f_1(\boldsymbol{x})=\sum_{i=1}^{D}|x_i-\exp((i/D)^2)/3|^{0.5},$

$$f_2(\boldsymbol{x})=\sum_{i=1}^{D}(x_i-0.5\cos(10\pi/D)-0.5)^2\},x_i\in[0,1]$$

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